Dividing Space: First Day Activity for Math Camp and for Linear Algebra (and other courses)

AMS Special Session on Math Circle Activities as a Gateway into Mathematics
Joint Mathematics Meetings, Boston MA, January 4, 2023
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Resource

Geometry by Discovery by David Gay,
John Wiley & Sons, NY NY, 1998.

Setting the stage:

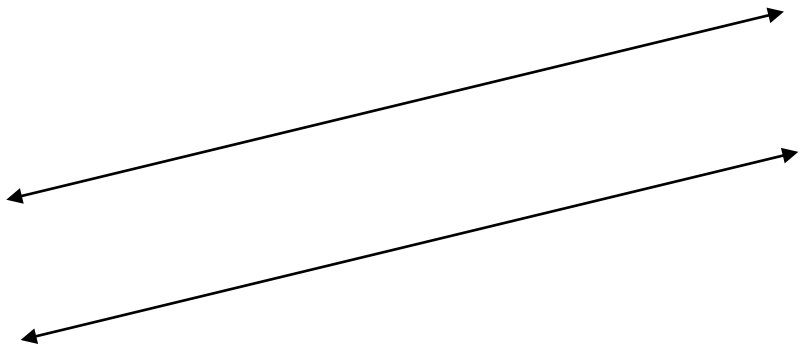
1. A point on a line divides a line into how many sections?
2. Two points on a line divide a line into how many sections?
3. Three points on a line divide a line into how many sections?
4. What can we claim? How certain are we?

Dividing a plane

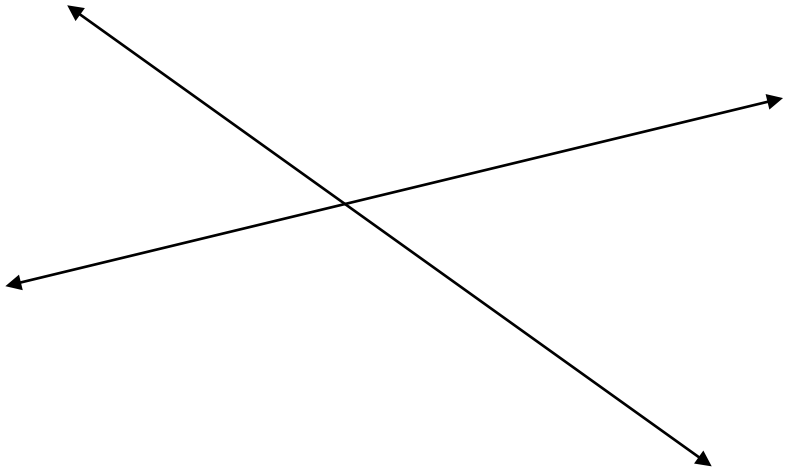
1. A line divides a plane into how many regions?
2. What about two lines?
3. Three lines?
4. Four lines?
5. Five?

Dividing a plane with two lines

Three regions if parallel.

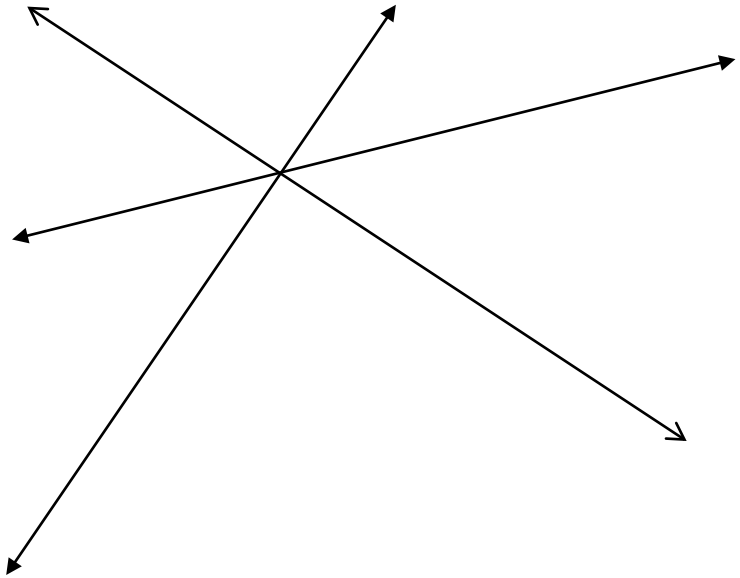


Four if they intersect.

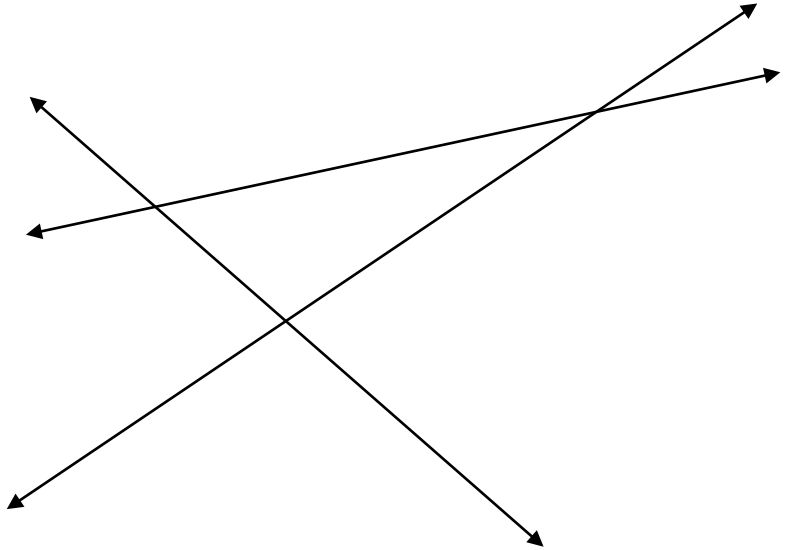


Three Lines dividing a plane

Intersecting in a single point

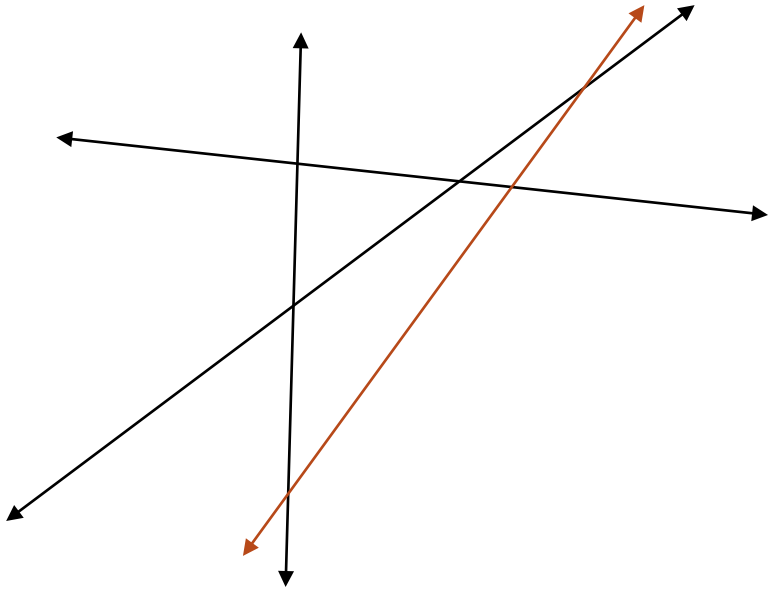


Pairwise intersections



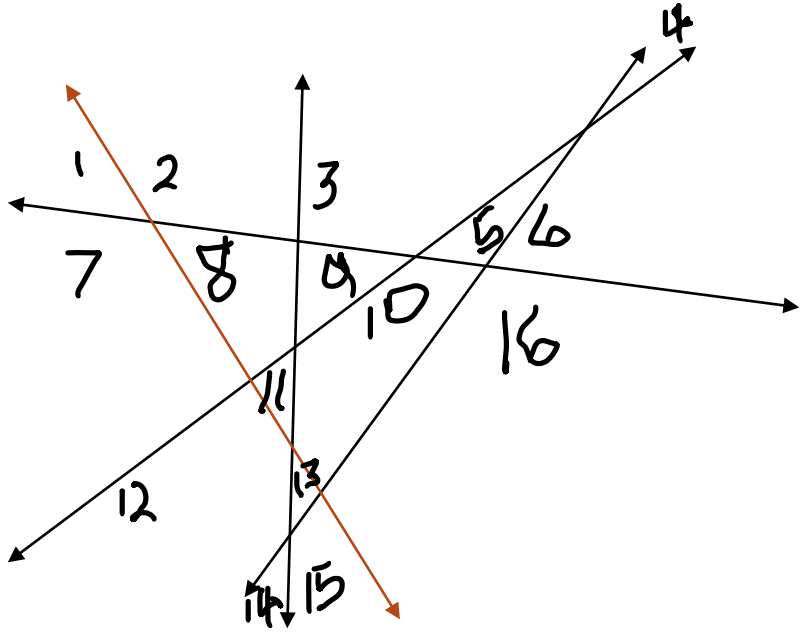
Maximize Regions with Four or more lines:

Four lines: 11 regions



Five lines:

16 regions



Place the new line so it creates as many intersections with existing lines as possible!

Pattern?

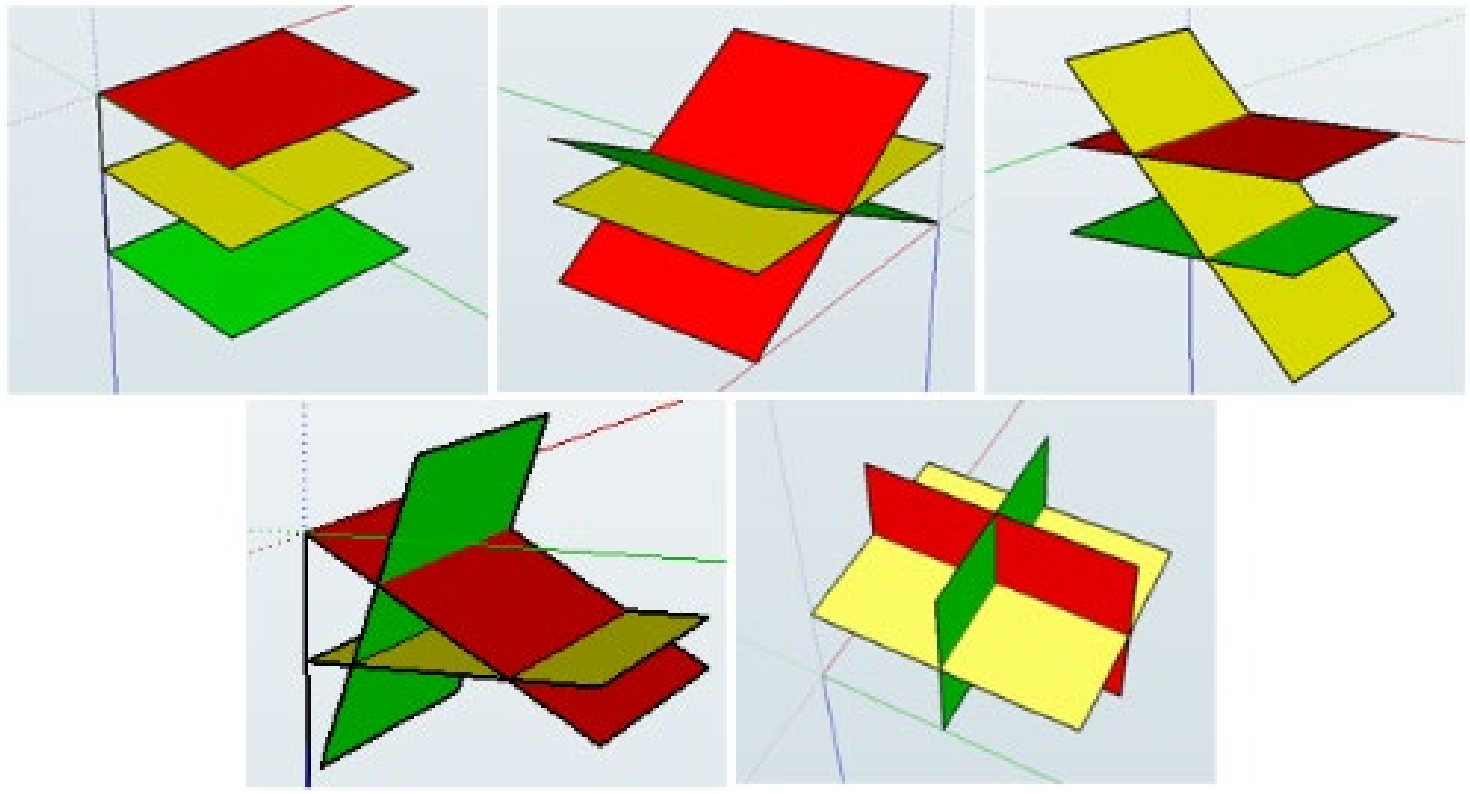
Lines	Maximum # of regions
0	1
1	2
2	4
3	7
4	11
5	16

- Add *n* to previous number.
- One more than triangular numbers.
- $\frac{n\cdot(n+1)}{2}+1$

Dividing 3-dimensional space

Explore the maximum number of regions that *n* planes divide 3-space into.

Dividing Space with 3 Planes



http://mathandmultimedia.com/2011/05/25/intersection-of-planes-google-sketchup/

Number of dividing items	By Points on a Line	By Lines in a Plane	By Planes in Space
0	1	1	1
1	2	2	2
2	3	4	4
3	4	7	8
4	5	11	?
5	6	16	
⋮	⋮	⋮	
n	$n + 1$	$\frac{n \cdot (n+1)}{2} + 1$	

Sequences, Recursion, and Linear Systems

$$s(n) = p(n-1) + s(n-1), s(0) = p(0) = 1$$

$$s(n) = \left(\frac{n(n-1)}{2} + 1\right) + s(n-1), s(0) = 1$$

$$s(n) = \frac{1}{6}(n^3 + 5n + 6)$$

$$\begin{matrix} a(1) + b(1^2) + c(1^3) = s(1) - 1 = 1 \\ a(2) + b(2^2) + c(2^3) = s(2) - 1 = 3 \\ a(3) + b(3^2) + c(3^3) = s(3) - 1 = 7 \end{matrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 3 \\ 3 & 9 & 27 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/6 \end{array}\right]$$

Undergraduate Course Foreshadowing

- Reasoning, Conjecture, and Proof
- Sequences, Series, Induction
- Visualization of Linear Spaces and Geometry
- Solving Linear Systems

More Information

- **Geometry by Discovery** by David Gay, John Wiley & Sons, NY NY, 1998.
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